Analytic Quest for Confining Interaction Kernels in Instantaneous Bethe–Salpeter Equations

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The Salpeter equation is a well-known three-dimensional approximation to the Bethe–Salpeter equation crucial for the description of bound states by quantum field theories. Therefore, it represents a standard tool in hadron physics. It is obtained by assuming the bound-state constituents to propagate as free particles (with effective masses) and by considering their interactions in the 'instantaneous limit'.

However, depending on the specific Lorentz behaviour of the Bethe–Salpeter interaction kernel, implementation of confinement in the Salpeter equation in too naïve ways might lead to the prediction of unstable states where only truly bound states are expected: Their energy eigenvalues may be embedded in a continuous spectrum, for instance.

These observations call for a rigorous spectral analysis of the Salpeter equation, in order to pin down its essential spectral features by regarding a bound state as stable if its energy (or mass) eigenvalue belongs to a real and discrete part of the spectrum that is bounded from below. Reusing ideas and methods designed in our previous studies of the 'reduced Salpeter equation' [1, 2] and of its improvement [3–5] derived by allowing for 'dressed' propagators of all bound-state constituents [6, 7], we therefore embark on a thorough investigation of full — in contrast to reduced — Salpeter equations for fermion–antifermion bound states.

We exploit the serendipity that describing confinement by a configuration-space potential of harmonic-oscillator form reduces the Salpeter integral equation to a system of (radial) eigenvalue differential equations. The analysis of confining interactions of time-component Lorentz-vector nature is comparatively easy and the stability of all bound states may be established analytically [8] by constructing operator inequalities and applying the characterization of all discrete eigenvalues of self-adjoint operators bounded from below by the minimum-maximum principle [9–14]. Regrettably, the proof cannot be transferred to the case of a confining interaction of Lorentz-scalar or -pseudoscalar structure but for a linear combination of time-component Lorentz-vector and Lorentz-scalar confining interactions stability is assured (irrespective of the relative sign of our two contributions) if the vector dominates the scalar [15]. 'Non-harmonically', we will face integral equations [16].

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